3. (Lab #7) Study & Verification of Maximum Power Transfer

Objective:

1- To measure power in a dc load.

2- To verify experimentally that the maximum power transferred by a dc source to a load occurs when the resistance of the load equals the resistance of the source.

Introduction:

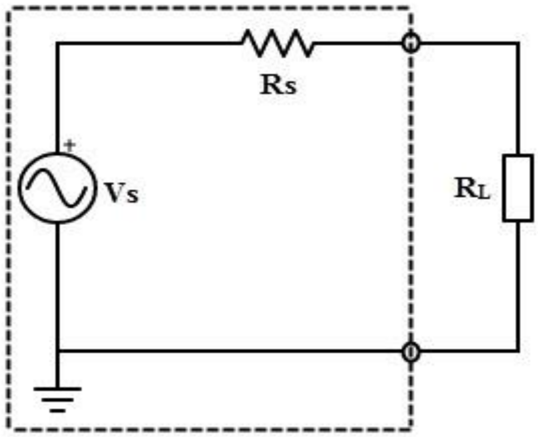
In any electric circuit, the electrical energy from the supply is delivered to the load where it is converted into a useful work. Practically, the entire supplied power will not present at load due to the heating effect and other constraints in the network. Therefore, there exist a certain difference between drawing and delivering powers.

The load size always affects the amount of power transferred from the supply source, i.e., any change in the load resistance results to change in power transfer to the load. Thus, the maximum power transfer theorem ensures the condition to transfer the maximum power to the load.

1) Maximum Power Transfer Theorem Statement

The maximum power transfer theorem states that in a linear, bilateral DC network, maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source.

If it is an independent voltage source, then its series resistance (internal resistance Rs) or if it is independent current source, then its parallel resistance (internal resistance Rs) must equal to the load resistance R_L to deliver maximum power to the load.



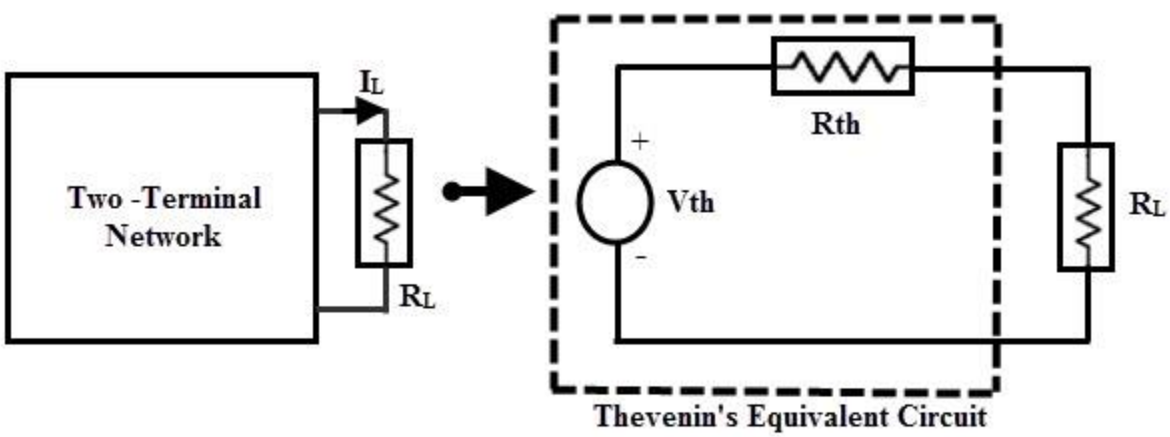
[Figure 7-1] Simple resistance circuit showing internal resistance

2) Proof of Maximum Power Transfer Theorem

The maximum power transfer theorem ensures the value of the load resistance, at which the maximum power is transferred to the load.

Consider the below DC two terminal network (left side circuit) , to which the condition for maximum power is determined , by obtaining the expression of power absorbed by load with use of mesh or nodal current methods and then make derivation of the resulting expression with respect to load resistance R_L.

But this is quite a complex procedure. But we have seen that the complex part of the network can be replaced with a Thévenin’s equivalent as shown below.



[Figure 7-2] Converting Complex Network to Thévenin’s Equivalent Circuit

The original two terminal circuit is replaced with a Thevenin's equivalent circuit across the variable load resistance. The current through the load for any value of load resistance is

$$I_L = \frac{V_{th}}{R_{th} + R_L} \qquad (7-1)$$

The power absorbed by the load is

$$P_L = I_L^2 \times R_L \qquad (7-2)$$

$$= \left[\frac{V_{th}}{R_{th} + R_L}\right]^2 \times R_L \qquad (7-3)$$

Form the above expression the power delivered depends on the values of R_{th} and R_L. However the Thévenin's equivalent is constant, the power delivered from this equivalent source to the load entirely depends on the load resistance R_L. To find the exact value of R_L, we apply differentiation to P_L with respect to R_L and equating it to zero as

$$\frac{dp(R_L)}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L \times (R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = 0 \qquad (7-4)$$

$\rightarrow (R_{Th} + R_L) - 2R_L = 0$

$\rightarrow R_L = R_{Th}$

Therefore, this is the condition of matching the load where the maximum power transfer occurs when the load resistance is equal to the Thévenin's resistance of the circuit. By substituting the $R_{th} = R_L$ in equation 1, we get

The maximum power delivered to the load is,

$$P_{max} = \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L \qquad Where\ R_{Th} = R_L \qquad (7-5)$$

$$= \frac{V_{Th}^2}{4R_{Th}} \qquad (7-6)$$

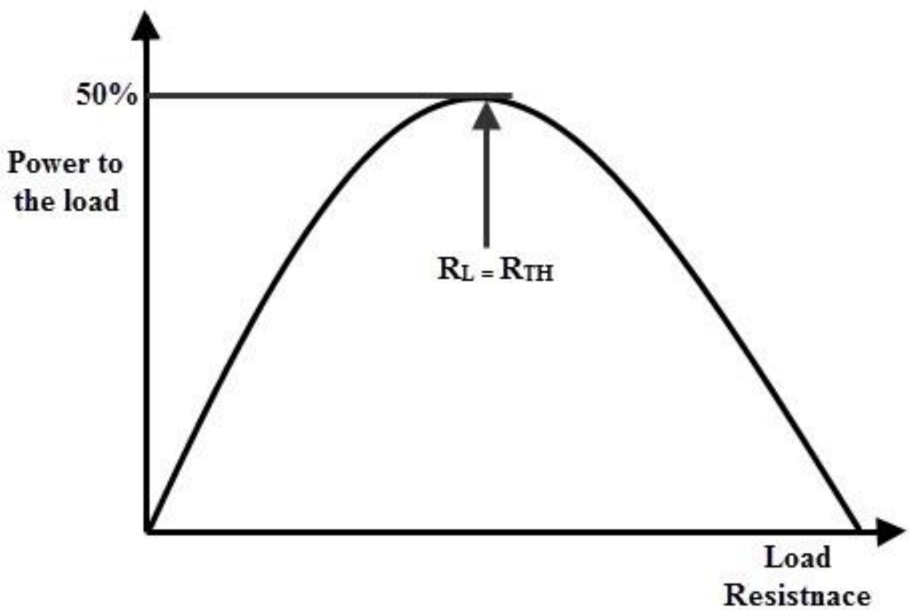
Total power transferred from source is

$$P_T = I_L^2(R_{Th} + R_L) \quad (7-7)$$

$$= 2I_L^2 R_L \quad (7-8)$$

Hence, the maximum power transfer theorem expresses the state at which maximum power is delivered to the load, that is , when the load resistance is equal to the Thévenin's equivalent resistance of the circuit. Below figure (7-3) shows a curve of power delivered to the load with respect to the load resistance.

Note that the power delivered is zero when the load resistance is zero as there is no voltage drop across the load during this condition. Also, the power will be maximum, when the load resistance is equal to the internal resistance of the circuit (or Thévenin's equivalent resistance). Again, the power is zero as the load resistance reaches to infinity as there is no current flow through the load.



[Figure 7-3] Curve of Power Delivered to the Load With Respect to the Load Resistance

3) Power Transfer Efficiency

We must remember that this theorem results maximum power transfer but not a maximum efficiency. If the load resistance is smaller than source resistance, power dissipated at the load is reduced while most of the power is dissipated at the source then the efficiency becomes lower.

Consider the total power delivered from source equation (equation (7-8)), in which the power is dissipated in the equivalent Thévenin's resistance R_{TH} by the voltage source V_{TH}.

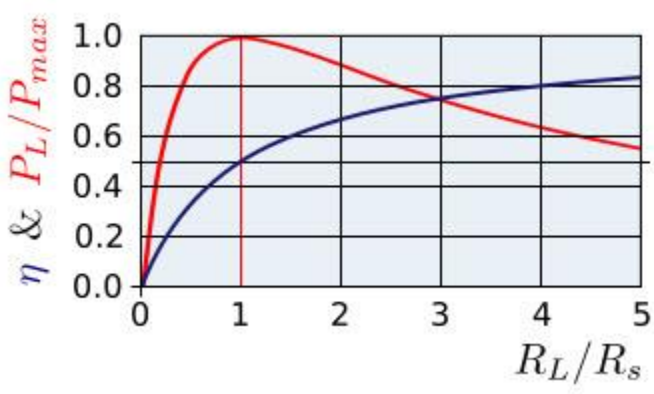
Therefore, the efficiency under the condition of maximum power transfer is

$$Efficiency = \frac{Output}{Input} \times 100 \qquad (7-9)$$

$$= \frac{I_L^2 R_L}{2 I_L^2 R_L} \times 100 \qquad (7-10)$$

$$= 50\%$$

Hence, at the condition of maximum power transfer, the efficiency is 50%, that means a half percentage of generated power is delivered to the load and at other conditions small percentage of power is delivered to the load , as indicated in efficiency verses maximum power transfer the curves below.



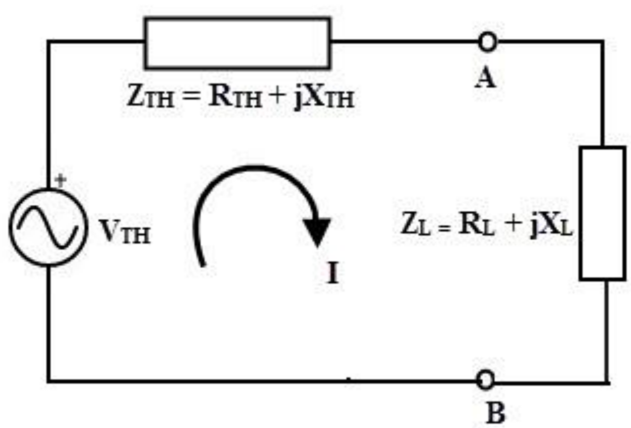
[Figure 7-4] Efficiency verses maximum power transfer

For some applications, it is desirable to transfer maximum power to the load than achieving high efficiency such as in amplifiers and communication circuits.

On the other hand, it is desirable to achieve higher efficiency than maximized power transfer in case of power transmission systems where a large load resistance (much larger value than internal source resistance) is placed across the load. Even though the efficiency is high the power delivered will be less in those cases.

4) Maximum Power Transfer Theorem for AC Circuits

It can be stated as in an active network, the maximum power is transferred to the load when the load impedance is equal to the complex conjugate of an equivalent impedance of a given network as viewed from the load terminals.



[Figure 7-5] Active network

Consider the above Thévenin's equivalent circuit across the load terminals in which the current flowing through the circuit is given as

$$I = \frac{V_{Th}}{Z_{Th} + Z_L} \qquad (7-11)$$

$$Where\ Z_L = R_L + jX_L$$

$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Therefore, I = V_{TH} / (R_L + jX_L + R_{TH} + jX_{TH}) \qquad (7-12)$$

$$= V_{Th} / ((R_L + R_{TH}) + j(X_L + X_{TH})) \qquad (7-13)$$

The power delivered to the load,

$$P_L = I^2 R_L \qquad (7-14)$$

$$P_L = V^2_{TH} \times R_L / ((R_L + R_{Th})2 + (XL + XTH)2) \qquad (7-15)$$

For maximum power the derivative of the above equation must be zero, after simplification we get

$$X_L + X_{Th} = 0 \qquad (7-16)$$

$$X_L = - X_{Th} \qquad (7-17)$$

Putting the above relation in equation (7-15), we get

$$P_L = V^2_{TH} \times \frac{R_L}{(R_L + R\text{Th})^2} \qquad (7-18)$$

Again for maximum power transfer, derivation of above equation must be equal to zero, after simplification we get

$$R_L + R_{TH} = 2 R_L \qquad (7-19)$$

$$R_L = R_{Th} \qquad (7-20)$$

Hence, the maximum power will transferred to the load from source, if $R_L = R_{Th}$ and $X_L = - X_{Th}$ in an AC circuit. This means that the load impedance should be equal to the complex conjugate of equivalent impedance of the circuit,

$$Z_L = Z_{Th} \qquad (7-21)$$

, where Z_{Th} is the complex conjugate of the equivalent impedance of the circuit.

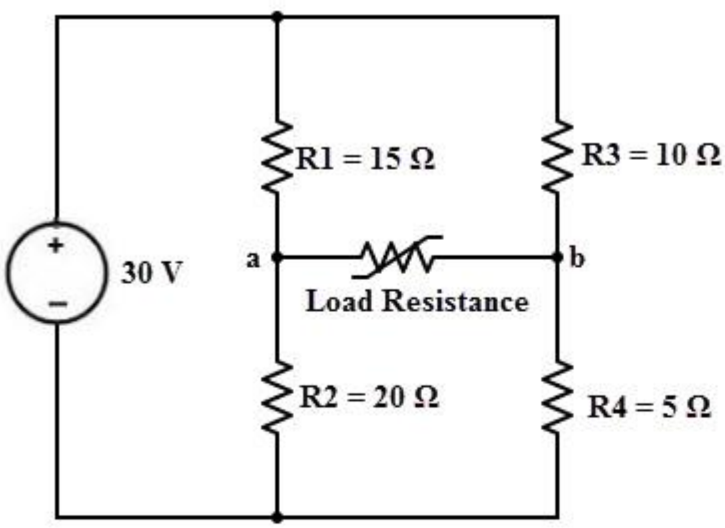
This maximum power transferred,

$$Pmax = V_{Th}^2 / 4\ R_{Th} \quad \text{or} \quad V_{Th}^2 / 4\ R_L \qquad (7-22)$$

Tasks for the (Lab #7): Study & Verification of Maximum Power Transfer

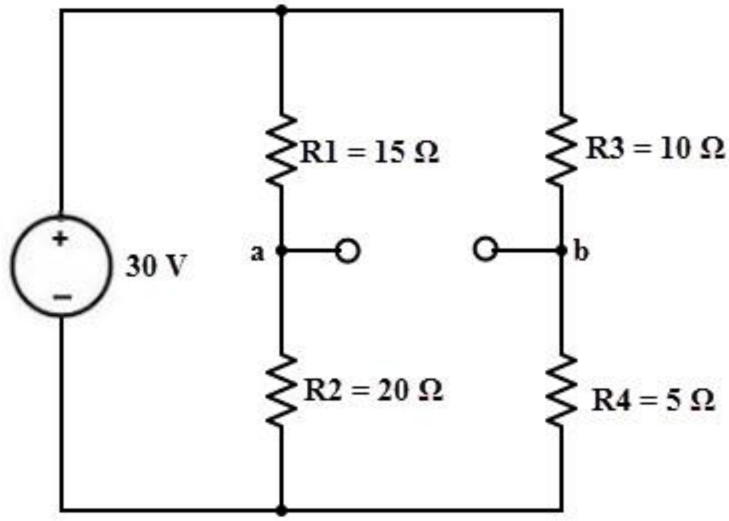
1) Understand Maximum Power Transfer Example to DC circuit

1. Consider the below circuit Figure (7-6) to which we determine the value of the load resistance that receives the maximum power from the supply source and the maximum power under the maximum power transfer condition.



[Figure 7-6] Applying Maximum Power Transfer Circuit

2. Disconnect the load resistance from the load terminals a and b. To represent the given circuit as Thévenin's equivalent, we are to determine the Thévenin's voltage V_{Th} and Thévenin's equivalent resistance R_{Th}.



[Figure 7-7] Applying Maximum Power Transfer Circuit Using Thévenin's Equivalent

The Thévenin's voltage or voltage across the terminals ab is $V_{ab} = V_a – V_b$

$$Va = V \times \frac{R2}{(R1 + R2)} \quad (7-23)$$

$$= 30 \times 20 /\times (20 + 15)$$

$$= 17.14\, V$$

$$Vb = V \times \frac{R4}{(R3 + R4)} \quad (7-24)$$

$$= 30 \times 5 /(10 + 5)$$

$$= 10\, V$$

$$Vab = 17.14 - 10$$

$$= 7.14\, V$$

$$VTH = Vab = 7.14\, Volts$$

3. Calculate the Thevenin's equivalent resistance R_{Th} by replacing sources with their internal resistances (here assume that voltage source has zero internal resistance so it becomes a short circuited).

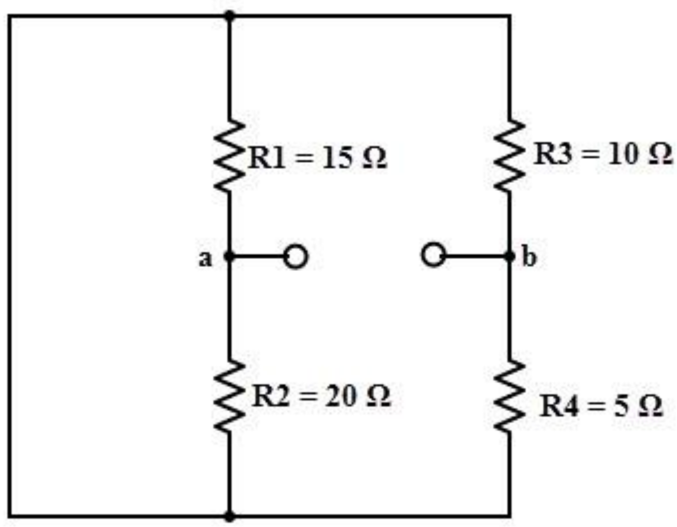
   Thevenin's equivalent resistance or resistance across the terminals ab is

$$R_{Th} = R_{ab} = [R1R2 / (R1 + R2)] + [R3R4 / (R3 + R4)]$$

$$= [(15 \times 20) / (15 + 20)] + [(10 \times 5) / (10 + 5)]$$

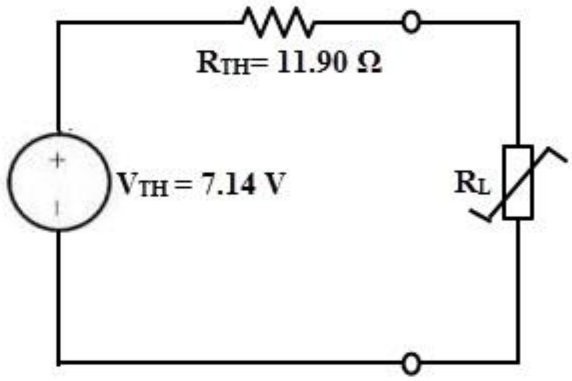
$$= 8.57 + 3.33$$

$$= 11.90 \; Ohms$$



[Figure 7-8] Applying Maximum Power Transfer Circuit Using Thévenin's Equivalent, internal resistance is short circuit

The Thévenin's equivalent circuit with above calculated values by reconnecting the load resistance is shown below.



[Figure 7-9] Thévenin's equivalent circuit

From the maximum power transfer theorem, R_L value must equal to the R_{Th} to deliver the maximum power to the load.

Therefore, $R_L = R_{Th} = 11.90\ Ohms$

And the maximum power transferred under this condition is,

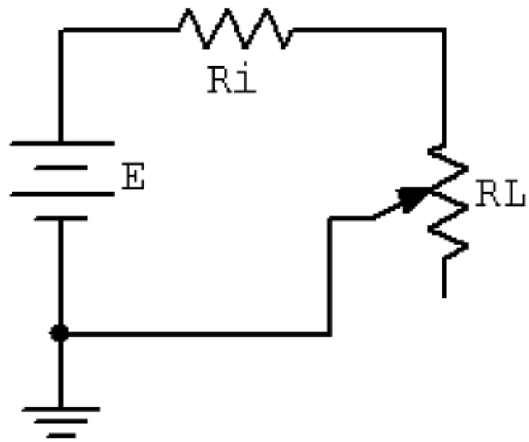
$$Pmax = V_{TH}^2 \,/\, 4\, R_{TH}$$
$$= (7.14)2 \,/\, (4 \times 11.90)$$
$$= 50.97 \,/\, 47.6$$
$$= 1.07\ Watts$$

2) Apply Maximum Power Transfer Example to DC circuit

Equipment & Materials

1. Adjustable DC Power Supply
2. Digital Multimeter
3. Resistor 3.3 kΩ
4. Rheostat

Schematic



[Figure 7-10] Maximum Power Transfer DC Circuit

Procedure

1. Consider the simple series circuit of Figure (7-10)) using E = 10 volts and R_i = 3.3 k. R_i forms a simple voltage divider with R_L. The power in the load is V_L^2 / R_L and the total circuit power is $E^2 / (R_i + R_L)$. The larger the value of R_L, the greater the load voltage, however, this does not mean that very large values of R_L will produce maximum load power due to the division by R_L. That is, at some point V_L^2 will grow more slowly than R_L itself. This crossover point should occur when R_L is equal to R_i. Further, note that as R_L increases, total circuit power decreases due to increasing total resistance. This should lead to an increase in efficiency. An alternate way of looking at the efficiency question is to note that as R_L increases, circuit current decreases. As power is directly proportional to the square of current, as RL increases the power in R_i must decrease leaving a larger percentage of total power going to R_L.

2. Using R_L = 30, compute the expected values for load voltage, load power, total power and efficiency, and record them in Table (7-1). Repeat for the remaining RL values in the Table. For the middle entry labeled Actual, insert the measured value of the 3.3kΩ used for R_i.

3. Connect the circuit of Figure (7-10) using E = 10 volts and R_i = 3.3 k. Use the rheostat for R_L and set it to 30Ω. Measure the load voltage and record it in Table (7-2). Calculate the load power, total power and efficiency, and record these values in Table (7-2). Repeat for the remaining resistor values in the table.

4. Create two plots of the load power versus the load resistance value using the data from the two tables, one for calculated and the other for experimental. For best results make sure that the horizontal axis (R_L) uses a log scaling instead of linear.

5. Create two plots of the efficiency versus the load resistance value using the data from the two tables, one for calculated and the other for experimental. For best results make sure that the horizontal axis (R_L) uses a log scaling instead of linear.

[Table 7-1] calculated Maximum Power Transfer

R_L	V_L	P_L	P_T	η
30				
150				
500				
1k				
2.5k				
Actual				
4K				
10K				
25K				
70K				
300K				

[Table 7-2] Experimental results for Maximum Power Transfer

R_L	V_L	P_L	P_T	η
30				
150				
500				
1k				
2.5k				
Actual				
4K				
10K				
25K				
70K				
300K				

Questions

1. At what point does maximum load power occur?

2. At what point does maximum efficiency occur?

3. Is it safe to assume that generation of maximum load power is always a desired goal? Why / why not?