Discrete Mathematics Study Center

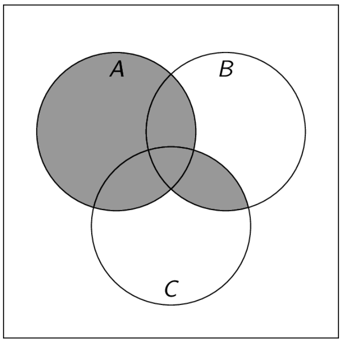
Membership Tables

We combine sets in much the same way that we combined propositions. Asking if an element x is in the resulting set is like asking if a proposition is true. Note that x could be in any of the original sets.

What does the set $A \cup (B \cap C)$ look like? We use 1 to denote the presence of some element x and 0 to denote its absence.

A	B	C	$B \cap C$	$A \cup (B \cap C)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

This is a **membership table**. It can be used to draw the Venn diagram by shading in all regions that have a 1 in the final column. The regions are defined by the left-most columns.



We can also use membership tables to test if two sets are equal. Here are two methods of showing if $\overline{A \cap B} = \overline{A} \cup \overline{B}$:

- Showing each side is a subset of the other:

$$\begin{aligned} x \in \overline{A \cap B} &\rightarrow x \notin A \cap B \\ &\rightarrow \neg(x \in A \cap B) \\ &\rightarrow \neg(x \in A \land x \in B) \\ &\rightarrow \neg(x \in A) \lor \neg(x \in B) \\ &\rightarrow x \notin A \lor x \notin B \\ &\rightarrow x \in \overline{A} \lor x \in \overline{B} \\ &\rightarrow x \in \overline{A} \cup \overline{B} \end{aligned}$$

$$\begin{aligned} x \in \overline{A} \cup \overline{B} &\rightarrow x \notin A \lor x \notin B \\ &\rightarrow \neg(x \in A) \lor \neg(x \in B) \\ &\rightarrow \neg(x \in A \land x \in B) \\ &\rightarrow \neg(x \in A \cap B) \\ &\rightarrow x \notin A \cap B \\ &\rightarrow x \in \overline{A \cap B} \end{aligned}$$

- Using membership tables:

A	B	C	$A \cap B$	$\overline{\mathbf{A} \cap \mathbf{B}}$	$\overline{A}$	$\overline{B}$	$\overline{\mathbf{A}} \cup \overline{\mathbf{B}}$
1	1	1	1	**0**	0	0	**0**
1	1	0	1	**0**	0	0	**0**
1	0	1	0	**1**	0	1	**1**
1	0	0	0	**1**	0	1	**1**
0	1	1	0	**1**	1	0	**1**
0	1	0	0	**1**	1	0	**1**
0	0	1	0	**1**	1	1	**1**
0	0	0	0	**1**	1	1	**1**

Since the columns corresponding to the two sets match, they are equal.

It is **not sufficient** to simply draw the Venn diagrams for two sets to show that they are equal: you need to show why your Venn diagram is correct (typically with a membership table).

There is an additional way to prove two sets are equal, and that is to use **set identities**. In the following list, assume A and B are sets drawn from a universe U.

- Identity Law: $A \cup \emptyset = A, A \cap U = A$
- Idempotent Law: $A \cup A = A, A \cap A = A$
- Domination Law: $A \cup U = U, A \cap \emptyset = \emptyset$
- Complementation Law: $\overline{\overline{A}} = A$
- Commutative Law: $A \cup B = B \cup A, A \cap B = B \cap A$
- Associative Law: $A \cup (B \cup C) = (A \cup B) \cup C, A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C), A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- Absorption Law: $A \cup (A \cap B) = A$ and $A \cap (A \cup B) = A$
- De Morgan's Law: $\overline{A \cap B} = \overline{A} \cup \overline{B}, \overline{A \cup B} = \overline{A} \cap \overline{B}$
- Complement Law: $A \cup \overline{A} = U, A \cap \overline{A} = \emptyset$
- Difference Equivalence: $A \setminus B = A \cap \overline{B}$

Note the similarities to logical equivalences! Here are some examples of how to determine if two sets are equal:

- Is $(A \setminus C) \cap (B \setminus C)$ equal to $(A \cap B) \cap \overline{C}$? First, we can use a membership table:

A	B	C	$A \setminus C$	$B \setminus C$	$(\mathbf{A} \setminus \mathbf{C}) \cap (\mathbf{B} \setminus \mathbf{C})$	$A \cap B$	$\overline{C}$	$(\mathbf{A} \cap \mathbf{B}) \cap \overline{\mathbf{C}}$
1	1	1	0	0	**0**	1	0	**0**
1	1	0	1	1	**1**	1	1	**1**
1	0	1	0	0	**0**	0	0	**0**
1	0	0	0	0	**0**	0	1	**0**
0	1	1	0	0	**0**	0	0	**0**
0	1	0	0	1	**0**	0	1	**0**
0	0	1	0	0	**0**	0	0	**0**
0	0	0	0	0	**0**	0	1	**0**

Since the columns corresponding to the two sets match, they are equal. We can also use set identities:

$$\begin{aligned}(A \setminus C) \cap (B \setminus C) &= (A \cap \overline{C}) \cap (B \cap \overline{C}) && \text{Difference Equivalence}\\ &= (A \cap B) \cap (\overline{C} \cap \overline{C}) && \text{Associative Law}\\ &= (A \cap B) \cap \overline{C} && \text{Idempotent Law}\end{aligned}$$

- Is $(A \setminus C) \cap (C \setminus B)$ equal to $A \setminus B$? Let's use some set identities:

$$\begin{aligned}(A \setminus C) \cap (C \setminus B) &= (A \cap \overline{C}) \cap (C \cap \overline{B}) && \text{Difference Equivalence}\\ &= (A \cap \overline{B}) \cap (C \cap \overline{C}) && \text{Associative Law}\\ &= (A \cap B) \cap \emptyset && \text{Complement Law}\\ &= \emptyset && \text{Domination Law}\end{aligned}$$

Note that, in general, $A \setminus B \neq \emptyset$ (\eg, let $A = \{1, 2\}, B = \{1\}$). Therefore, these sets are not equal. (Note the similarity to finding truth settings that invalidate an argument!)